数理解析研究所講究録
243巻 1975年 42-66

ON $\tilde{H}$-COBORDISMS BETWEEN THREE DIMENSIONAL HOMOLOGY HANDLES

AKIO KAWAUCHI

Graduate school, Osaka City University

The present note will introduce a cobordism theory, called $\tilde{H}$-cobordism, to the class of 3-dimensional homology oriented handles and to the class of 3-dimensional homology non-orientable handles. These classes modulo $\tilde{H}$-cobordism relations will form groups $\Omega(S^1 \times S^2)$, $\Omega(S^1 \times_\tau S^2)$, called $\tilde{H}$-cobordism groups, respectively.

We will discuss about the properties of the invariants of $\Omega(S^1 \times S^2)$ and $\Omega(S^1 \times_\tau S^2)$. Then we will know that $\Omega(S^1 \times S^2)$ is so related to the Fox-Milnor's classical knot cobordism group C^1 and the Levine's matrix cobordism goup G_-, and that $\Omega(S^1 \times_\tau S^2)$ is isomorphic to the direct sum of infinite countable copies of the cyclic group of order 2.

Section 1 will construct the oriented $\tilde{H}$-cobordism group $\Omega(S^1 \times S^2)$. In Section 2, we will discuss about the properties of the invariants of $\Omega(S^1 \times S^2)$ and compare $\Omega(S^1 \times S^2)$ with the Fox-Milnor's knot cobordism group C^1 and with the Levine's matrix cobordism group G_-. Section 3 will describe the non-orientable $\tilde{H}$-cobordism group $\Omega(S^1 \times_\tau S^2)$ and determine its group structure. Section 4 will propose further discussions and questions.

Throughout this note, the space will be considered in the piecewise-linear category.

§ 1

A CONSTRUCTION OF THE ORIENTED $\tilde{H}$-COBORDISM GROUP $\Omega(S^1 \times S^2)$

A 3-dimensional <u>homology orientable handle</u> M is a compact 3-manifold having the homology of the orientable handle $S^1 \times S^2$: $H_*(M;Z) \approx H_*(S^1 \times S^2;Z)$. A homology orientable handle M is <u>oriented</u> if one generator of $H_3(M;Z)$ is distinguished. The class of all homology oriented handles is denoted by $\vec{\mathcal{C}}(S^1 \times S^2)$. If M is in $\vec{\mathcal{C}}(S^1 \times S^2)$, then $-M$, which is the same manifold as M but has the opposite orientation, also lies in $\vec{\mathcal{C}}(S^1 \times S^2)$.

<u>1.1 DEFINITION</u>. Two homology oriented handles M_0 , M_1 are <u>$\tilde{H}$-cobordant</u> and denoted by $M_0 \sim M_1$, if there exists a compact connected oriented 4-manifold W with the boundary ∂W being the disjoint union $M_0 \cup (-M_1)$ and such that there is an infinite cyclic <u>connected</u> covering $(\tilde{W};\tilde{M}_0,\tilde{M}_1) \longrightarrow (W;M_0,M_1)$ of the triad $(W;M_0, M_1)$ with $H_*(\tilde{W};Q)$ being finitely generated over Q.

As usual, the triad $(W;M_0,M_1)$ is called an <u>$\tilde{H}$-cobordism</u>.

For any $M \in \vec{\mathcal{C}}(S^1 \times S^2)$, note that $H_*(\tilde{M};Q)$ is finitely generated over Q. Then the use of the Mayer-Vietoris sequence clearly yields the following.

<u>1.2 LEMMA</u>. <u>The $\tilde{H}$-cobordism relation</u> $\sim$ <u>is an equivalence relation</u>.

If $M \sim S^1 \times S^2$ then we write $M \sim 0$. Note that $M \sim 0$ if and only if there exists a compact connected oriented 4-manifold W with the boundary ∂W being M and such that there is an infinite cyclic connected covering $(\tilde{W},\tilde{M}) \longrightarrow (W,M)$ of the pair (W,M) with $H_*(\tilde{W};Q)$ being finitely generated over Q. In this case the notation $(W;M,\emptyset)$ may be adapted.

1.3 DEFINITION. The set $\Omega(S^1\times S^2)$ is defined to be the set of $\vec{\mathcal{C}}(S^1\times S^2)$ modulo the $\tilde{H}$-cobordism relation.

For any $M\in\vec{\mathcal{C}}(S^1\times S^2)$ $[M]$ denotes the element of $\Omega(S^1\times S^2)$ having M as the representative.

To show that the set $\Omega(S^1\times S^2)$ forms a non-trivial abelian group, we introduce a sum oparation, called a circle union.

Let M_0, $M_1\in\vec{\mathcal{C}}(S^1\times S^2)$ and choose polyhedral simple closed curves $\omega_0\subset M_0$, $\omega_1\subset M_1$ which represent generators of $H_1(M_0;Z)$, $H_1(M_1;Z)$, respectively. Then there exist closed connected orientable surfaces $F_0\subset M_0$, $F_1\subset M_1$ such that $F_0\cap\omega_0$, $F_1\cap\omega_1$ consist of single points, respectively.[To see this, first note that the identity map $\omega_0\subset\omega_0$ can be extended to a piecewise-linear map $f_0:M_0\longrightarrow\omega_0$ by means of the elementary obstruction theory. Second, note that there is a point $p_0\in\omega_0$ such that the preimage $f_0^{-1}(p_0)$ is a closed (not necessarily connected) orientable surface. Now choose the component of $f_0^{-1}(p_0)$ containing p_0 as F_0. Similarly, the desired F_1 exists.]

Consider the solid torus $S^1\times B^2$ and piecewise-linear embeddings

$$h_0:S^1\times B^2\times 0 \longrightarrow M_0$$
$$h_1:S^1\times B^2\times 1 \longrightarrow M_1$$

such that

(1) there exist points $s \in S^1, b \in \mathrm{Int}B^2$ with $h_0(s \times B^2 \times 0) \subset F_0$, $h_0(S^1 \times b \times 0) = \omega_0$, $h_1(s \times B^2 \times 1) \subset F_1$ and $h_1(S^1 \times b \times 1) = \omega_1$,

(2) the orientations of $S^1 \times B^2 \times 0$ and $S^1 \times B^2 \times 1$ are induced from some orientation of $S^1 \times B^2 \times [0,1]$ and h_0 is orientation-preserving and h_1 is orientation-reversing.

<u>1.4 DEFINITION</u>. The oriented manifold

$$M_0 \circ M_1 = M_0 \cup_{h_0} S^1 \times B^2 \times [0,1] \cup_{h_1} M_1 - S^1 \times \mathrm{Int}B^2 \times [0,1]$$

is called a <u>circle union</u> of M_0 and M_1.

From construction, easily we have $M_0 \circ M_1 \in \vec{\mathcal{C}}(S^1 \times S^2)$.

<u>1.5 REMARK</u>. In general, the homeomorphism type of $M_0 \circ M_1$ depends upon the choices of ω_0 and ω_1. For example let $\omega \subset S^1 \times S^2$ be a simple closed curve of geometrical index 1 and $T(\omega)$ be the tubular neighborhood of ω in $S^1 \times S^2$. If the circle union $S^1 \times S^2 \circ S^1 \times S^2$ is defined to be the double of $\mathrm{cl}(S^1 \times S^2 - T(\omega))$, then $S^1 \times S^2 \circ S^1 \times S^2$ is clearly piecewise-linearly homeomorphic to $S^1 \times S^2$. On the other hand, consider for example a simple closed curve $\omega' \subset S^1 \times S^2$ of geometrical index 3 and algebraic index 1(See figure 1.) and let $T(\omega')$ be the tubular neighborhood of ω' in $S^1 \times S^2$. If the circle union $S^1 \times S^2 \circ' S^1 \times S^2$ is defined to be the double of $\mathrm{cl}(S^1 \times S^2 - T(\omega'))$, then $S^1 \times S^2 \circ' S^1 \times S^2$ is not homeomorphic to $S^1 \times S^2 \approx S^1 \times S^2 \circ S^1 \times S^2$, because the natural inclusion $\partial T(\omega') \longrightarrow S^1 \times S^2 \circ' S^1 \times S^2$ induces the monomorphism $\pi_1(\partial T(\omega')) \longrightarrow \pi_1(S^1 \times S^2 \circ' S^1 \times S^2)$ by the loop theorem and hence $\pi_1(S^1 \times S^2 \circ' S^1 \times S^2) \not\approx Z$.

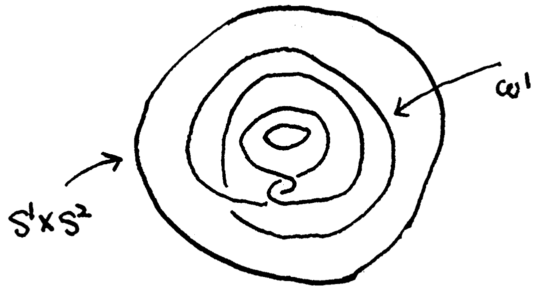


figure 1.

In spite of REMARK 1.5, for arbitrary two circle unions $M_0 \bigcirc M_1$, $M_0 \bigcirc' M_1$ we have the following.

1.6 LEMMA. $M_0 \bigcirc M_1 \sim M_0 \bigcirc' M_1$.

Proof. Let $M_0 \bigcirc M_1 = M_0 \times 0 \cup_{h_0} S^1 \times B^2 \times [0,1] \cup_{h_1} M_1 \times 0 - S^1 \times \mathrm{Int} B^2 \times [0,1]$ and $-(M_0 \bigcirc' M_1) = M_0 \times 1 \cup_{h_0'} S^1 \times B^2 \times [0,1] \cup_{h_1'} M_1 \times 1 - S^1 \times \mathrm{Int} B^2 \times [0,1]$ and

$$W = M_0 \times [0,1] \begin{matrix} \cup_{h_0} & S^1 \times B^2 \times [0,1] & \cup_{h_1} \\ \cup_{h_0'} & S^1 \times B^2 \times [0,1] & \cup_{h_1'} \end{matrix} M_1 \times [0,1] \text{ (See figure 2.).}$$

Clearly we have $\partial W = M_0 \bigcirc M_1 \cup -(M_0 \bigcirc' M_1)$. Further, the infinite cyclic connected covering $\widetilde{M_0 \bigcirc M_1} \longrightarrow M_0 \bigcirc M_1$ can be easily extended to an infinite cyclic covering $\widetilde{W} \longrightarrow W$. From construction the restriction to $M_0 \bigcirc' M_1$ gives the infinite cyclic connected covering $-\widetilde{M_0 \bigcirc' M_1} \longrightarrow -(M_0 \bigcirc' M_1)$. Using the Mayer-Vietoris sequence we obtain that $H_*(\widetilde{W};Q)$ is finitely generated over Q. Thus, the triad $(W; M_0 \bigcirc M_1, M_0 \bigcirc' M_1)$ gives an $\tilde{H}$-cobordism and the proof is completed.

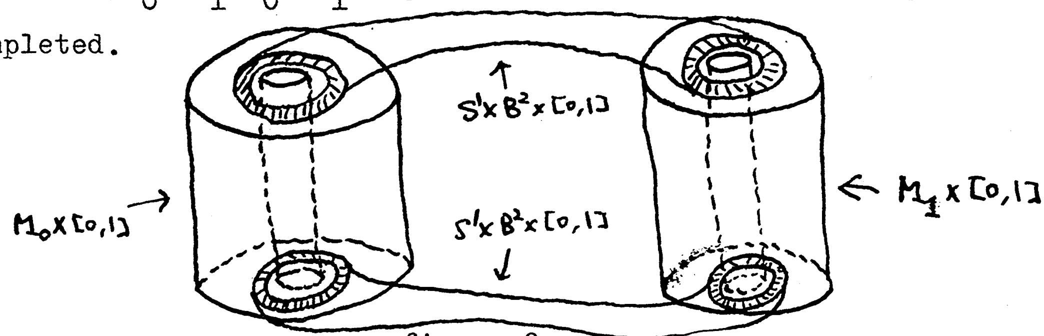


figure 2.

<u>1.7 LEMMA</u>. $M_0 \sim M_1$ <u>is equivalent to</u> $M_0 \bigcirc -M_1 \sim 0$.

Proof. Assume $M_0 \sim M_1$. Then there is a compact connected oriented 4-manifold W with $\partial W = M_0 \cup -M_1$ and such that for some infinite cyclic connected covering $(\widetilde{W};\widetilde{M}_0,\widetilde{M}_1) \longrightarrow (W;M_0,M_1)$ $H_*(\widetilde{W};Q)$ is finitely generated over Q. Let $M_0 \bigcirc -M_1 = M_0 \cup_{h_0} S^1 \times B^2 \times [0,1] \cup_{h_1} (-M_1) - S^1 \times \mathrm{Int} B^2 \times [0,1]$ and $W' = W \cup_{h_0,h_1} S^1 \times B^2 \times [0,1]$ (See figure 3.). Clearly, $\partial W' = M_0 \bigcirc -M_1$ and the infinite cyclic covering $\widetilde{W} \longrightarrow W$ is extended to an infinite cyclic covering $\widetilde{W}' \longrightarrow W'$ and $H_*(\widetilde{W}';Q)$ is finitely generated over Q. Therefore $M_0 \bigcirc -M_1 \sim 0$. Conversely, assume $M_0 \bigcirc -M_1 \sim 0$. Then there is a compact connected oriented 4-manifold W'' with $\partial W'' = M_0 \bigcirc -M_1$ and such that for some infinite cyclic connected covering $(\widetilde{W}'', \widetilde{M_0 \bigcirc -M_1}) \longrightarrow (W'', M_0 \bigcirc -M_1)$, $H_*(\widetilde{W}'';Q)$ is finitely generated over Q. Note that by the definition of the circle union there is a natural injection $j: S^1 \times \partial B^2 \times [0,1] \to M_0 \bigcirc -M_1$. Now we let $W''' = W'' \cup_j S^1 \times B^2 \times [0,1]$. It is easy to see that $\partial W'''$ is equal to the disjoint union $M_0 \cup -M_1$ and that the triad $(W''';M_0,M_1)$ gives an $\widetilde{H}$-cobordism between M_0 and M_1. This completes the proof.

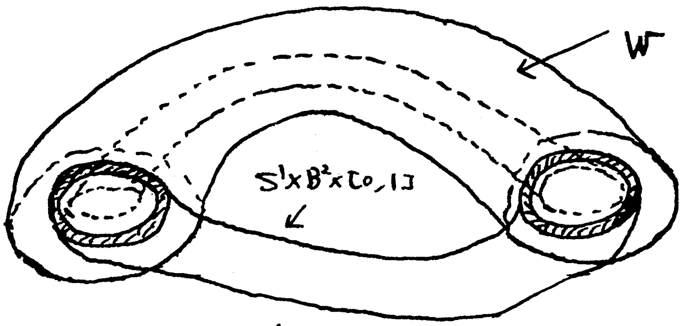


figure 3.

<u>1.8 LEMMA</u>. <u>If</u> $M_0 \sim 0$ <u>and</u> $M_1 \sim 0$, <u>then</u> $M_0 \bigcirc M_1 \sim 0$.

Proof. Let $(W;M_0,\emptyset)$, $(W';M_1,\emptyset)$ be $\tilde{H}$-cobordisms, and let $M_0 \circ M_1 = M_0 \cup_{h_0} S^1\times B^2\times[0,1] \cup_{h_1} M_1 - S^1\times \mathrm{Int}B^2\times[0,1]$. If we let $W'' = W \cup_{h_0} S^1\times B^2\times[0,1] \cup_{h_1} W'$ (See figure 4.), then the triad $(W'';M_0\circ M_1,\emptyset)$ gives an $\tilde{H}$-cobordism, which completes the proof.

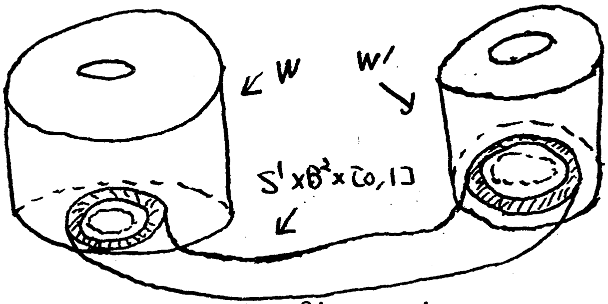


figure 4.

Now we state the main theorem of this section.

<u>1.9 THEOREM</u>. <u>The set $\Omega(S^1\times S^2)$ forms an abelian group under the sum $[M_0] + [M_1] = [M_0\circ M_1]$. The zero element of this group is $[S^1\times S^2]$. The inverse of any element $[M]$ is the element $[-M]$.</u>

Proof. To show that the sum $[M_0] + [M_1] = [M_0\circ M_1]$ is well-defined, we let $M_0 \sim M_0'$ and $M_1 \sim M_1'$. By LEMMA 1.7, $M_0\circ -M_0' \sim 0$ and $M_1\circ -M_1' \sim 0$. Then by LEMMA 1.8 $(M_0\circ -M_0')\circ(M_1\circ -M_1') \sim 0$. On the other hand, clearly

$$(M_0\circ M_1)\circ -(M_0'\circ M_1') \sim (M_0\circ -M_0')\circ(M_1\circ -M_1')$$

(Use LEMMA 1.6.). Hence again by LEMMA 1.7 $M_0\circ M_1 \sim M_0'\circ M_1'$. Thus, $[M_0] = [M_0']$ and $[M_1] = [M_1']$ imply $[M_0]+[M_1] = [M_0']+[M_1']$. It is clear that $([M_0]+[M_1])+[M_2] = [M_0]+([M_1]+[M_2])$ and $[M_0]+[M_1] = [M_1]+[M_0]$. Also, we have $[M]+[S^1\times S^2]=[M\circ S^1\times S^2]=[M]$ and, by LEMMA 1.7, $[M]+[-M] = [S^1\times S^2]$. This completes the proof.

The group $\Omega(S^1\times S^2)$ is called the (<u>oriented</u>) <u>$\tilde{H}$-cobordism</u>

group between 3-dimensional homology oriented handles. The zero element is denoted by 0 and the inverse of $[M]$ is $-[M]$.

§ 2

GEOMETRIC AND ALGEBRAIC STRUCTURES OF $\Omega(S^1 \times S^2)$

Let $p: \tilde{M} \longrightarrow M$ be the infinite cyclic orientation-preserving covering projection and t be a generator of the covering transformation group. Since M is oriented, one fundamental class $[M] \in H_3(M;Z)$ is specified. Then the choice of t determines a finite fundamental class $\mu \in H_2(\tilde{M};Z)(\approx Z)$ of $\tilde{M}$. In fact, we let $\mu = p_*^{-1}(\omega \cap [M])$, where $\omega \in H^1(M;Z)$ is a cocycle identified with the covering transformation t and $p_*: H_2(\tilde{M};Z) \approx H_2(M;Z)(\approx Z)$ (See Kawauchi [6,Remark 2.4].). Note that the dual isomorphism $\cap\mu: H^1(\tilde{M};Q) \approx H_1(\tilde{M};Q)$ holds(See Kawauchi [5,Theorem 2.3].). Equivalently, the skew-symmetric cup product pairing

$$H^1(\tilde{M};Q) \times H^1(\tilde{M};Q) \xrightarrow{\cup} H^2(\tilde{M};Q) \underset{\approx}{\overset{\cap\mu}{}} H_0(\tilde{M};Q) = Q$$

is non-singular (See Milnor[10,p127].).

2.1 DEFINITION. The bilinear form $<\ ,\ >: H^1(\tilde{M};Q) \times H^1(\tilde{M};Q) \longrightarrow Q$ defined by the equality $<x,y> = (x \cup ty)\cap\mu + (y \cup tx)\cap\mu$ is called the quadratic form of M.

It is easy to check that the quadratic form is uniquely determined by the oriented M (in particular, it does not depend upon the choice of t), and that it is a symmetric bilinear form with the identity $\langle tx,ty\rangle = \langle x,y\rangle$. Furthermore, this form $\langle\ ,\ \rangle$ is always non-singular. To see this, it is convenient to introduce the concept of the Alexander polynomial.

2.2 DEFINITION. The (rational) Alexander polynomial $A(t)$ of M is the characteristic polynomial of the isomorphism $t : H_1(\tilde{M};Q) \longrightarrow H_1(\tilde{M};Q)$ (See Kawauchi[6] for details and the definition of the integral Alexander polynomial.).

$A(t)$ is the invariant of M up to units $ct^i \in Q[t,t^{-1}]$ and has the properties $A(\pm 1) \neq 0$ and $A(t) \doteq A(t^{-1})$.

Suppose , for all x, $\langle x,y\rangle = [x \cup (t-t^{-1})y] \cap \mu = 0$. Then $(t-t^{-1})y = -t^{-1}(t-1)(t+1)y = 0$, which implies $y = 0$, since $A(\pm 1) \neq 0$. Thus, the quadratic form $\langle\ ,\ \rangle$ is non-singular.

Since $\langle\ ,\ \rangle$ is non-singular and symmetric, with a suitable basis of $H_1(\tilde{M};Q)$, $\langle\ ,\ \rangle$ represents a rational diagonal matrix

$$\begin{pmatrix} a_1 & & & & & \\ & \ddots & & & 0 & \\ & & a_m & & & \\ & & & -b_1 & & \\ & 0 & & & \ddots & \\ & & & & & -b_n \end{pmatrix} \qquad (a_i > 0,\ b_j > 0).$$

2.3 DEFINITION. The integer $\sigma(M) = m - n$ is called the signature of the oriented M.

It is not difficult to see that $\sigma(M)$ is even (, since $\dim_Q H_1(\tilde{M};Q) = \deg A(t)$ is even) and that $\sigma(-M) = -\sigma(M)$.

2.4 THEOREM. If $M \sim 0$, then the signature $\sigma(M)$ is 0 and the Alexander polynomial $A(t)$ of M has the form $f(t)f(t^{-1})$ for some rational polynomial $f(t)$.

One may note an analogy of the signature and the Alexander polynomial between the oriented $\widetilde{H}$-cobordism group $\Omega(S^1 \times S^2)$ and the Fox-Milnor's knot cobordism group C^1(See Fox-Milnor [2].). This relation will be clarified in this section.

2.5 DEFINITION. The reduced Alexander polynomial $\widetilde{A}(t)$ of M is the rational polynomial obtained from the Alexander polynomial $A(t)$ by cancelling the factors of the type $f(t)f(t^{-1})$.

Let $\widetilde{A}(t)$, $\widetilde{A}'(t)$ be the reduced Alexander polynomials of M, M', respectively. The following is a direct consequence of THEOREM 2.4.

2.6 THEOREM. If $M \sim M'$, then $\sigma(M) = \sigma(M')$ and $\widetilde{A}(t) \doteq \widetilde{A}'(t)$. [Note that there is a canonical isomorphism $H_1(\widetilde{M \cup -M'}) \approx H_1(\widetilde{M}) + H_1(-\widetilde{M}')$.]

2.7 PROOF OF THEOREM 2.4. Since $M \sim 0$, there exists an $\widetilde{H}$-cobordism $(W;M,\emptyset)$. Then for an infinite cyclic connected covering $(\widetilde{W},\widetilde{M}) \longrightarrow (W,M)$, $H_*(\widetilde{W};Q)$ is finitely generated over Q.

Now we consider the following diagram

$$\begin{tikzcd}
H^1(\widetilde{W};Q) \arrow[r, "i^*"] & H^1(\widetilde{M};Q) \arrow[r, "\delta"] \arrow[d, "\cap\mu"] & H^2(\widetilde{W},\widetilde{M};Q) \arrow[d, "\cap\bar{\mu}"] \\
 & H_1(\widetilde{M};Q) \arrow[r, "i_*"] & H_1(\widetilde{W};Q)
\end{tikzcd}$$

Here, the top sequence is exact and the vertical maps are isomorphisms and $\bar{\mu} \in H_3(\tilde{W},\tilde{M};Z)(\approx Z)$ is a finite fundamental class (See Kawauchi [5,Theorem 2.3].) obtained from the finite fundamental class $\mu \in H_2(\tilde{M};Z)$ by the boundary-isomorphism $\partial: H_3(\tilde{W},\tilde{M};Z) \approx H_2(\tilde{M};Z)$. And the square is commutative.

Because the sequence $0 \longrightarrow \mathrm{Im}\, i^* \longrightarrow H^1(\tilde{M};Q) \longrightarrow \mathrm{Im}\,\delta \longrightarrow 0$ is exact, the equality $A(t) \doteq B(t)C(t)$ holds, where $B(t),C(t)$ are the characteristic polynomials of the linear isomorphisms $t\colon \mathrm{Im}\, i^* \longrightarrow \mathrm{Im}\, i^*$ and $t\colon \mathrm{Im}\,\delta \longrightarrow \mathrm{Im}\,\delta$, respectively.

By the commutativity of the above sequare, we have the isomorphism $\cap\bar{\mu} : \mathrm{Im}\,\delta \longrightarrow \mathrm{Im}\, i_*$. This asserts that the equality $C(t^{-1}) \doteq B(t)$ holds.[Use the identities $(tu)\cap\bar{\mu} = t^{-1}(u\cap\bar{\mu})$ and $\mathrm{Im}\, i^* = \mathrm{Hom}(\mathrm{Im}\, i_*,Q)$.] Thus, we have $A(t) \doteq C(t)C(t^{-1}) \doteq B(t^{-1})B(t)$.

Next, for all $u \in H^1(\tilde{W};Q)$, suppose $\langle i^*(u),y\rangle = 0$. This situation is equivalent to $\delta(t-t^{-1})y = 0$, that is, $(t-t^{-1})y \in \mathrm{Im}\, i^*$, because $\langle i^*u,y\rangle = i^*u \cup (t-t^{-1})y = u \cup \delta(t-t^{-1})y$.[Use the above square is commutative.] Using $(t-t^{-1})\mathrm{Im}\, i^* \subset \mathrm{Im}\, i^*$ and the isomorphism $t-t^{-1}: H^1(\tilde{M};Q) \longrightarrow H^1(\tilde{M};Q)$, $(t-t^{-1})y \in \mathrm{Im}\, i^*$ is equivalent to $y \in \mathrm{Im}\, i^*$. Thus, we showed that the orthogonal complement of $\mathrm{Im}\, i^*$ is $\mathrm{Im}\, i^*$ itself. Then, a familiar process implies $\sigma(M) = 0$ (See for example Milnor-Husemoller[11,p13].). This completes the proof.

Let $\mathcal{K}$ be the set of knot types of tame oriented 1-knots in the oriented S^3. We shall construct a function $m : \mathcal{K} \longrightarrow \vec{\mathcal{C}}(S^1 \times S^2)$. (Now we regard the class $\vec{\mathcal{C}}(S^1 \times S^2)$ as the set of orientation-preserving homeomorphism types of homology oriented handles.)

Let $T(k) \subset S^3$ be the tubular neighborhood of a knot $k \subset S^3$. By Schubert [12], the knotted torus $T(k)$ in S^3 has unique meridean and longitude curves (up to isotopies of $\partial T(k)$). Define $m(k)$ to be the oriented manifold obtained from the surgery of S^3 along $T(k)$ by using the unique meridean and longitude curves: $m(k) = S^3 - T(k) \cup B^2 \times S^1$.(The orientation of $m(k)$ will adapt the orientation induced from $S^3-T(k)$.)

This assignment clearly implies a function m:*) $\mathcal{K} \longrightarrow \vec{\mathcal{C}}(S^1 \times S^2)$ from the knot types to the homeomorphism types.

Two knot types $\mathfrak{k}_1, \mathfrak{k}_2$ are (knot) cobordant if for representative knots $k_1 \in \mathfrak{k}_1$ and $k_2 \in \mathfrak{k}_2$ the sum $k_1 \# -k_2 \subset S^3$ bounds a locally flat 2-cell in the 4-cell B^4. Such a concept is called the knot cobordism. The set $\mathcal{K}$ modulo the knot cobordism relation forms an abelian group C^1, called the knot cobordism group (See Fox-Milnor [2] for details.).

Note that the function $m: \mathcal{K} \longrightarrow \vec{\mathcal{C}}(S^1 \times S^2)$ induces a homomorphism $m: C^1 \longrightarrow \Omega(S^1 \times S^2)$. In fact, easily we have $m(k_1 \# k_2) = m(k_1) \circ m(k_2)$, and if k is cobordant to a trivial knot then $m(k) \sim 0$ [To see this, let $D^2 \subset B^4$ be a locally flat 2-cell with $k = \partial D^2 \subset S^3$. By using an embedding $\varphi: \partial B^2 \times B^2 \longrightarrow S^3$, giving a tubular neighborhood of $k = \varphi(\partial B^2 \times p)$ such that a circle $\varphi(\partial B^2 \times p')$ is the longitude curve, we construct a 4-manifold $W = B^4 \cup_\varphi B^2 \times B^2$. Then $\partial W = m(k)$ and a 2-sphere $\Sigma = D^2 \cup B^2 \times p \subset W$ is locally flat. By performing a surgery along the tubular neighborhood of Σ, we obtain a 4-manifold W' with $\partial W' = m(k)$

*) This function is not injective. A non-invertible knot would provide such an example.

and $H_*(W';Z) \approx H_*(S^1;Z)$. The triad $(W';m(k),\emptyset)$ gives an $\tilde{H}$-cobordism.].

2.8 LEMMA. <u>The homomorphism</u> $m\colon C^1 \longrightarrow \Omega(S^1 \times S^2)$ <u>satisfies</u> $\sigma\langle k\rangle = \sigma[m(k)]$ <u>and</u> $A_{\langle k\rangle}(t) \doteq A_{[m(k)]}(t)$ <u>for all</u> $\langle k\rangle \in C^1$.

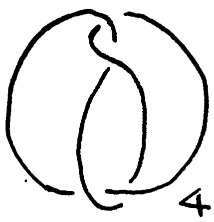
The proof will be given later.

By LEMMA 2.8, the known results of C^1 also imply the following two corollaries.

2.9 COROLLARY. <u>For any integer</u> i, <u>there exists</u> $M \in \mathcal{C}(S^1 \times S^2)$ <u>with</u> $\sigma(M) = 2i$.

2.10 COROLLARY. <u>The oriented $\tilde{H}$-cobordism group $\Omega(S^1 \times S^2)$ has the free part of infinite rank and contains a torsion element.</u>

For example, for the figure eight knot 4_1(See figure 5.), the element $[m(4_1)] \in \Omega(S^1 \times S^2)$ gives an èlement of order 2, because the element $\langle 4_1\rangle \in C^1$ has order 2 and the reduced Alexander polynomial of $m(4_1)$ is t^2-3t+1 which implies $[m(4_1)] \neq 0$ by THEOREM 2.4.



4_1

figure 5.

Now we need a concept of Seifert matrices. A <u>Seifert matrix</u> V is an integral square matrix with $\det(V-V') = \pm 1$. (V' is the transepose of V.)

For two Seifert matrices V and W, if the block sum $V \oplus -W$ is congruent (over the integers) to a matrix of the form $\begin{pmatrix} 0 & B \\ C & D \end{pmatrix}$ (B,C,D are square matrices of the same size.) then V is said to be (<u>matrix</u>) <u>cobordant</u> to W. Such a concept is called the <u>matrix cobordism</u>. The set of Seifert matrices modulo the matrix cobordism relation forms a group G_- , called the <u>matrix cobordism group</u> (See Levine[8] for details. Note that only Seifert matrices with sign -1 are considered.). By Levine [9], G_- is isomorphic to the direct sum $\sum_{i=1}^{\infty} Z^i \oplus \sum_{i=1}^{\infty} Z_2^i \oplus \sum_{i=1}^{\infty} Z_4^i$.

For a knot k in S^3, denote $M(k)$ to be the knot exterior, or the closed knot complement of k, and $\tilde{M}(k)$ to be its infinite cyclic covering space. By a <u>Seifert matrix of the knot</u> k we will mean a Seifert matrix which is S-equivalent to a Seifert matrix associated with a Seifert surface of k. (See Trotter[13] for recent results of S-equivalences.)

The <u>quadratic form</u> $\langle\ ,\ \rangle: H^1(\tilde{M}(k),\partial\tilde{M}(k);Q) \times H^1(\tilde{M}(k),\partial\tilde{M}(k);Q) \longrightarrow Q$ of the oriented knot k in the oriented S^3 is defined by the equality $\langle x,y\rangle = (xUty)\cap\mu + (yUtx)\cap\mu$ (See Milnor [10] and Erle [1].), which is a complete analogue of DEFINITION 2.1.[Note that since k and S^3 are oriented, both t and μ are specified uniquely.] (Here, $\mu \in H_2(\tilde{M}(k),\partial\tilde{M}(k);Z)$ is a finite fundamental class [5].)

Erle [1] then showed that , <u>with a suitable basis of</u>

$H^1(\tilde{M}(k),\partial\tilde{M}(k);Q)$, <u>the linear isomorphism</u> $t : H^1(\tilde{M}(k),\partial\tilde{M}(k);Q) \longrightarrow H^1(\tilde{M}(k),\partial\tilde{M}(k);Q)$ <u>represents the matrix</u> $V'^{-1}V$ <u>and the quadratic form</u> $< , >$ <u>represents the matrix</u> $V + V'$ <u>for some non-singular Seifert matrix</u> V.

The same assertion also applies for the homology oriented handles.

By Kawauchi [5, Corollary 1.3], there is a piecewise-linear map $f : M \longrightarrow S^1$ such that $F = f^{-1}(p)$ is a closed conneted surface. Clearly, the homology class $[F] \in H_2(\tilde{M};Z)$ coincides with $\pm\mu \in H_2(\tilde{M};Z)$. If t is specified, then μ is also specified and hence we may orient F so that $[F] = \mu$. Let M^* be a manifold obtained from M by splitting along F. Note that a duality $H^1(F;Z) \approx H_1(M^*;Z)$ holds. Let $\partial M^* = F \cup -F$ (Here we identify the component of ∂M^* with the orientation compatible with F.). With dual bases of $H_1(F;Z)$ and $H_1(M^*;Z)$, the canonical homomorphism $H_1(F;Z) \longrightarrow H_1(M^*;Z)$ represents a square matrix V_0. To show that V_0 is a Seifert matrix, let V_0^- be another matrix representing the canonical homomorphism $H_1(-F;Z) \longrightarrow H_1(M^*;Z)$. By an analogy of Levine [7] it is not difficult to see that the matrix $tV_0 - V_0^-$ is a relation matrix of $H_1(\tilde{M};Z)$ and that V_0^- is in fact the transpose V_0' of V_0. Thus, $tV_0 - V_0'$ is a relation matrix of $H_1(\tilde{M};Z)$. Using $H_1(M;Z) = Z$, $\det(V_0 - V_0') = \pm 1$. Thus, V_0 is a Seifert matrix.

<u>2.11 DEFINITION</u>. A Seifert matrix V which is S-equivalent to V_0 is called a <u>Seifert matrix of</u> M (<u>with a specified generator of</u> $H_1(M;Z)$).

Note that if another generator of $H_1(M;Z)$ is specified then the transpose V' of V is considered as a Seifert matrix of M.

A technique of Erle [1] then implies the following :

2.12 LEMMA. With a suitable basis of $H^1(\tilde{M};Q)$, the linear isomorphism $t : H^1(\tilde{M};Q) \longrightarrow H^1(\tilde{M};Q)$ represents the matrix $V'^{-1}V$ and the quadratic form $< , >: H^1(\tilde{M};Q) \times H^1(\tilde{M};Q) \longrightarrow Q$ represents the matrix $V + V'$ for some non-singular Seifert matrix V of M.

Using LEMMA 2.12, we obtain a well-defined homomorphism $\psi : \Omega(S^1 \times S^2) \longrightarrow G_-$ sending homology oriented handles to the Seifert matrices (See Levine [9,p101].). [Note that the Seifert matrix V is always cobordant to the transpose V', although V is in general not S-equivalent to V' (See Trotter[13].).]

Thus, we sketched the following.

2.13 THEOREM. There is the commutative triangle

$$\begin{tikzcd} C^1 \arrow[rr, "m"] \arrow[dr, "\phi"'] & & \Omega(S^1 \times S^2) \arrow[dl, "\psi"] \\ & G_- & \end{tikzcd}$$

$$G_- = \sum_{i=1}^{\infty} Z^i \oplus \sum_{i=1}^{\infty} Z_2^i \oplus \sum_{i=1}^{\infty} Z_4^i$$

, where $\phi : C^1 \longrightarrow G_-$ is a canonical epimorphism defined by Levine [8] and $\psi : \Omega(S^1 \times S^2) \longrightarrow G_-$ is an epimorphism defined as above and m satisfies $\tilde{A}_{<k>}(t) \doteq \tilde{A}_{[m(k)]}(t)$ and $\sigma<k> = \sigma[m(k)]$ for all $\langle k\rangle \in C^1$.

2.14 PROOF OF LEMMA 2.7. The inclusion map $i : \tilde{M}(k) \longrightarrow \tilde{m}(k)$ induces an isomorphism $i_*: H_1(\tilde{M}(k);Q) \approx H_1(\tilde{m}(k);Q)$. From this, it follows that $A_k(t) \doteq A_{m(k)}(t)$, and hence $\tilde{A}_{\langle k\rangle}(t) \doteq \tilde{A}_{[m(k)]}(t)$.

Next, since the following triangle

$$\begin{tikzcd} H^1(\tilde{M}(k);Q) \times H^1(\tilde{M}(k);Q) \arrow[d, "\approx"', "i_* \times i_*"] \arrow[dr] & \\ H^1(\tilde{m}(k);Q) \times H^1(\tilde{m}(k);Q) \arrow[r] & Q \end{tikzcd}$$

is commutative, we obtain that $\sigma(k) = \sigma(m(k))$. This completes the proof.

The general problem of deciding a geometrical condition of $\tilde{H}$-cobordism seems difficult, but a partial result is presented here.

2.15 THEOREM. If $M \in \vec{\mathcal{C}}(S^1 \times S^2)$ is embeddable in a homology 4-sphere $\bar{S}^4$, then M is $\tilde{H}$-cobordant to 0.

Proof. Assume $M \subset \bar{S}^4$. By an easy computation of the homology, we obtain that M separates $\bar{S}^4$ into two manifolds, say, W_1, W_2 and that one of W_1, W_2 has the homology of the circle, say, $H_*(W_1;Z) \approx H_*(S^1;Z)$. Then $(W_1;M,\emptyset)$ gives an $\tilde{H}$-cobordism. This proves THEOREM 2.14.

Here are a few examples, whose somewhat analogous properties were also noticed by Kato[4].

2.16 EXAMPLES. First we consider a trefoil 3_1 (figure 6).

Using $\sigma(m(3_1)) = \pm 2$ or $\tilde{A}(t) = t^2-t+1$, we see that $m(3_1)$ is not $\tilde{H}$-cobordant to 0. Hence $m(3_1)$ is not embeddable to the 4-sphere S^4 and the minimal embedding dimension of $m(3_1)$ into a sphere is five [In fact, Hirsch[3] showed that every compact orientable manifold is locally flatly embeddable to the 5-sphere.].

figure 6.

On the other hand, a stevedore's knot 6_1 (figure 7) is a slice knot and hence $m(6_1) \sim 0$.

Note that a slice knot k can be realized as a local knot type of a 2-sphere $S(k)$ in S^4 with only one locally knotted point (See Fox-Milnor[2].).

Let $N(S(6_1);S^4)$ be the regular neighborhood of $S(6_1)$ in S^4. It is not hard to see that $\partial N(S(6_1);S^4) = m(6_1)$. Thus, $m(6_1)$ is embeddable in the 4-sphere S^4.

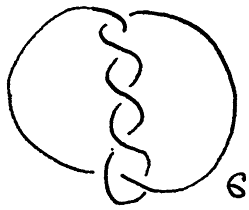
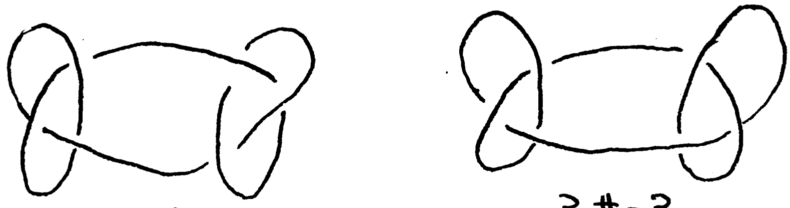


figure 7.

Similar arguments also applies for a granny knot $3_1\#3_1$ and a square knot $3_1\#-3_1$ (See figure 8.). In fact, $m(3_1\#3_1)$ is not embeddable to S^4, although $m(3_1\#-3_1)$ is embeddable to S^4, since $\sigma(m(3_1\#3_1)) = 2\sigma(3_1) = \pm 4$ and $3_1\#-3_1$ is a slice knot.



$3_1\#3_1$ $3_1\#-3_1$

figure 8.

§ 3
THE NON-ORIENTABLE $\tilde{H}$-COBORDISM GROUP $\Omega(S^1\times_\tau S^2)$

A 3-dimensional homology non-orientable handle M is a compact 3-manifold having the homology of the non-orientable handle $S^1\times S^2$: $H_*(M;Z)\approx H_*(S^1\times_\tau S^2;Z)$, and let $\mathcal{C}(S^1\times_\tau S^2)$ be the class of the homology non-orientable handles.

In $\mathcal{C}(S^1\times_\tau S^2)$, an $\tilde{H}$-cobordism relation is defined as an analogy of DEFINITION 1.1.

<u>3.1 DEFINITION</u>. Two homology handles M_0, M_1 in $\mathcal{C}(S^1\times_\tau S^2)$ are <u>$\tilde{H}$-cobordant</u> and denoted by $M_0 \sim M_1$ if there exists a compact connected (non-orientable) 4-manifold W with the boundary ∂W being the disjoint union $M_0 \cup M_1$ and such that there is an infinite cyclic connected covering $(\tilde{W};\tilde{M}_0,\tilde{M}_1) \longrightarrow (W;M_0,M_1)$ with $\tilde{W}$ being orientable and with $H_*(\tilde{W};Q)$ being finitely generated over Q. [Note that $\tilde{M}$ is always orientable (See Kawauchi[6, Lemma 2.3].).]

We say that M is $\tilde{H}$-cobordant to 0 if M is $\tilde{H}$-cobordant to $S^1\times_\tau S^2$.

For $M_0, M_1 \in \mathcal{C}(S^1\times_\tau S^2)$, choose polyhedral simple closed curves $\omega_0 \subset M_0$, $\omega_1 \subset M_1$ which represent generators of $H_1(M_0;Z)$, $H_1(M_1;Z)$, respectively. It is not difficult to see that the tubular neighborhoods $T(\omega_0) \subset M_0$ of ω_0 and $T(\omega_1) \subset M_1$ of ω_1 are both piecewise-linear homeomorphic to the solid Klein bottle $S^1\times_\tau B^2$.

Let $F_0 \subset M_0$, $F_1 \subset M_1$ be closed connected orientable surfaces transversally intersecting ω_0, ω_1 in single points, respectively.

Consider two piecewise-linear embeddings

$$h_0 : S^1\times_\tau B^2\times 0 \longrightarrow M_0$$
$$h_1 : S^1\times_\tau B^2\times 1 \longrightarrow M_1$$

such that there exist points $s\in S^1$, $b \in \mathrm{Int}B^2$ with $h_0(S^1\times_\tau b\times 0) = \omega_0$, $h_0(s\times_\tau B^2\times 0) \subset F_0$, $h_1(S^1\times_\tau b\times 1) = \omega_1$ and $h_1(s\times_\tau B^2\times 1) \subset F_1$.

As an analogy of DEFINITION 1.4, we may have DEFINITION 3.2.

3.2 DEFINITION. The homology non-orientable handle

$$M_0 \circ M_1 = M_0 \cup_{h_0} S^1\times_\tau B^2\times[0,1] \cup_{h_1} M_1 - S^1\times_\tau \mathrm{Int}B^2\times[0,1]$$

is called a circle union of M_0 and M_1.

It is not difficult to check that for two circle unions $M_0 \circ M_1$, $M_0 \circ' M_1$, $M_0 \circ M_1 \sim M_0 \circ' M_1$. Further, we can prove that $M_0 \sim M_1$ if and only if $M_0 \circ M_1 \sim 0$ as an analogy of LEMMA 1.7.

Thus, we sketched that the set $\Omega(S^1\times_\tau S^2) = \mathcal{C}(S^1\times_\tau S^2)/\sim$

forms an abelian group under the sum $[M_0] + [M_1] = [M_0 \bigcirc M_1]$. This group is called the non-orientable $\tilde{H}$-cobordism group of 3-dimensional homology non-orientable handles.

Every non-zero element of $\Omega(S^1 \times_\tau S^2)$ has order 2, by construction.

Further, $\Omega(S^1 \times_\tau S^2)$ is not finitely generated. Actually, the following is obtained.

3.3 THEOREM. $\Omega(S^1 \times_\tau S^2) \approx \sum_{i=1}^{\infty} Z_2^i$.

To prove THEOREM 3.3, the Alexander polynomial is useful.

The Alexander polynomial $A(t)$ of $M \in \mathcal{C}(S^1 \times_\tau S^2)$ is simply defined to be the characteristic polynomial of the linear isomorphism $t : H_1(\tilde{M}; Q) \longrightarrow H_1(\tilde{M}; Q)$. (See Kawauchi [6].)

Then THEOREM 3.3 follows from LEMMA 3.4 (, which is somewhat analogous to THEOREM 2.4).

3.4 LEMMA. *If $M \in \mathcal{C}(S^1 \times_\tau S^2)$ is $\tilde{H}$-cobordant to 0 then the Alexander polynomial $A(t)$ of M has a type of $f(t)f(-t^{-1})$ for some rational polynomial $f(t)$.*

3.5 PROOF OF THEOREM 3.3. By Kawauchi [6], the irreducible integral polynomial $A_n(t) = nt^2+t-n$ ($n = 1, 2, 3, \ldots.$) is realized as the Alexander polynomial of some $M_n \in \mathcal{C}(S^1 \times_\tau S^2)$. Then it is easy to see that $M_1, M_2, M_3, \ldots..$ represent a set of linearly independent elements of $\Omega(S^1 \times_\tau S^2)$. This completes the proof.

3.6 PROOF OF LEMMA 3.3. Since $M \sim 0$, there exists a compact connected 4-manifold W with $\partial W = M$ and such that for some infinite cyclic connected covering $(\tilde{W},\tilde{M}) \longrightarrow (W,M)$, $\tilde{W}$ is orientable and $H_*(\tilde{W};Q)$ is finitely generated over Q. Then from the exact sequence $H^1(\tilde{W};Q) \xrightarrow{i^*} H^1(\tilde{M};Q) \xrightarrow{\delta} H^2(\tilde{W},\tilde{M};Q)$ we obtain the short exact sequence $0 \longrightarrow \operatorname{Im} i^* \longrightarrow H^1(\tilde{M};Q) \longrightarrow \operatorname{Im}\delta \longrightarrow 0$. Thus we have $A(t) \doteq B(t)C(t)$, where $B(t)$, $C(t)$ are the characteristic polynomials of $t : \operatorname{Im} i^* \longrightarrow \operatorname{Im} i^*$, $t:\operatorname{Im}\delta \longrightarrow \operatorname{Im}\delta$, respectively. Since the square

$$\begin{array}{ccc} H^1(\tilde{M};Q) & \xrightarrow{\delta} & H^2(\tilde{W},\tilde{M};Q) \\ \approx\downarrow\cap\mu & & \approx\downarrow\cap\bar{\mu} \\ H_1(\tilde{M};Q) & \xrightarrow{i_*} & H_1(\tilde{W};Q) \end{array}$$

is commutative, we obtain the dual isomorphism $\cap\bar{\mu}:\operatorname{Im}\delta \approx \operatorname{Im} i_*$. Using the identities $(tu)\cap\bar{\mu} = -t^{-1}(u\cap\bar{\mu})$ and $\operatorname{Im} i^* = \operatorname{Hom}(\operatorname{Im} i_*,Q)$, this dual isomorphism gives the equality $C(-t^{-1}) \doteq B(t)$. This proves LEMMA 3.3.

§ 4

FURTHER DISCUSSIONS AND QUESTIONS

The most basic and interesting problem on this paper is the following question.

4.1 QUESTION. Whether or not are the homomorphisms m, ϕ, φ in THEOREM 2.13 isomorphic ?

This question also asks the difference between $\tilde{H}$-cobordism and

H-cobordism.

Usually , for compact closed oriented **n**-manifolds N_1^n, N_2^n, if there exists an oriented compact (n+1)-manifold H^{n+1} with $\partial H^{n+1} = N_1^n \cup -N_2^n$ and $H_*(H^{n+1}, N_1^n; Z) = 0$ $(= H_*(H^{n+1}, -N_2; Z))$, then N_1 is said to be <u>H-cobordant</u> to N_2. Also, such a concept is called <u>H-cobordism</u>.

<u>4.2 QUESTION.In</u> $\vec{\mathcal{C}}(S^1 \times S^2)$, <u>are H-cobordism and $\tilde{H}$-cobordism strictly distinct</u> ?

For example, it is not difficult to see that two H-cobordant homology oriented handles are $\tilde{H}$-cobordant.

In $\vec{\mathcal{C}}(S^1 \times S^2)$ or a class of more general manifolds it seems difficult to define a non-trivial H-cobordism group. However, for the class of homology oriented n-spheres, the H-cobordism group $\mathcal{H}(S^n)$ is defined in the natural way. In the piecewise-linear category, it is not so hard to see that $\mathcal{H}(S^n) = 0$ for $n \geq 5$. At $n = 4$, the author does not know whether $\mathcal{H}(S^4)$ vanishes or not. At $n = 3$, Kato pointed out that $\mathcal{H}(S^3)$ is non-trivial, that is, there exists a homology 3-sphere which is not the boundary of any homology 4-ball. In fact, the dodecahedral space $\bar{S}^3 = S^3/SL(2,5)$ is such an example.

<u>4.3 QUESTION</u>. <u>For any homology 3-sphere</u> $\bar{S}^3$, <u>is the connected sum</u> $S^1 \times S^2 \# \bar{S}^3$ <u>$\tilde{H}$-cobordant to</u> $S^1 \times S^2$?

Note that for the dodecahedral space $\bar{S}^3$, $S^1 \times S^2 \# \bar{S}^3$ is not H-cobordant to $S^1 \times S^2$.

Let $\vec{\mathcal{C}}^{3,n}$ be the class of compact oriented 3-manifolds having the integral homology of the connected sum $\#^n S^1 \times S^2$ of n copies of $S^1 \times S^2$. Similarly, let $\vec{\mathcal{C}}_Q^{3,n}$ be the class of compact oriented 3-manifolds having the rational homology of $\#^n S^1 \times S^2$.

<u>4.4 QUESTION</u>. <u>In</u> $\vec{\mathcal{C}}^{3,n}$ <u>or</u> $\vec{\mathcal{C}}_Q^{3,n}$ $(n \geq 2)$, <u>can a $\tilde{H}$-cobordism theory be developed</u> ?

It seems that for $n \geq 2$ all things would become extremely difficult.

In $\vec{\mathcal{C}}_Q^{3,1}$, the $\tilde{H}$-cobordism group $\Omega_Q^{3,1}$ is actually defined as an analogy of $\Omega(S^1 \times S^2)$.(This group is so related to the Levine's rational matrix cobordism group G_-^Q.)

Now suppose the $\tilde{H}$-cobordism groups $\Omega^{3,n} = \vec{\mathcal{C}}^{3,n}/\sim$ and $\Omega_Q^{3,n} = \vec{\mathcal{C}}_Q^{3,n}/\sim$ are obtained. Let $\Omega^{3,0} = \mathcal{H}(S^3)$ and let $\Omega_Q^{3,0}$ be the rational H-cobordism group of rational homology 3-spheres. The direct sums $\Omega^3 = \Omega^{3,0} \oplus \Omega^{3,1} \oplus \Omega^{3,2} \oplus \ldots..$ and $\Omega_Q^3 = \Omega_Q^{3,0} \oplus \Omega_Q^{3,1} \oplus \Omega_Q^{3,2} \oplus \ldots...$ would have ring structures under the connected sum operation.

In the higer dimensional case, we can also define the $\tilde{H}$-cobordism groups $\Omega(S^1 \times S^{n-1})$ and $\Omega(S^1 \times_\tau S^{n-1})$ of n-dimensional homology oriented and non-orientable handles, respectively.

The following seems not so difficult for $n \geq 5$.

<u>4.5 QUESTION</u>. <u>Is $\Omega(S^1 \times S^{n-1})$ isomorphic to the piecewise-linear (n-2)-knot cobordism group</u> C_{PL}^{n-2} ? <u>Also, is</u> $\Omega(S^1 \times_\tau S^{n-1})$ <u>isomorphic to</u> $\sum_{i=1}^{\infty} Z_2^i$ <u>if</u> n <u>is even, or to</u> 0 <u>if</u> n <u>is odd</u> ?

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